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The North Sea Problem III

Influence of a stationary windfield upon a bay

with an exponentially increasing depth

by

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1. Introduction

In this paper which is a generalization and a continuation of the preceding paper we shall study the influence of a stationary linear windfield upon a rectangular bay which has a variable depth. It will be assumed in particular that the depth is given by the exponential law (2.1). In that respect this model is a much better representation of the North Sea than the model considered in the preceding paper. This is demonstrated in figure 2.1 in which the depth profile of a longitudinal cross section of the North Sea is given.

The treatment follows closely that of the preceding paper. In order to facilitate the references the same notations will be used. Also in the numerical application the same values will be taken. In the present model the depth varies from 33 m at the "Dutch" coast to about 158 m at the ocean boundary. The mean harmonic depth h_m is then 65 m which equals the uniform depth of the model of the preceding paper.

The elevation at the "Dutch" coast due to the linear windfield (2.7) is given by table 6.4 and is graphically illustrated in figure 6.1. In order to estimate the influence of the rotation of the Earth the elevation has also been calculated for $\Omega = 0$. The elevation at the middle of the "Dutch" coast is given below for the various cases.

Exponential depth, $\Omega \neq 0$

$$(1.1) \quad \pi a^{-1} g h_m \zeta(\tfrac{1}{2}a, 0) = 1.67\ U_0 + 0.71\ U_1 + 1.43\ U_2 +$$
$$-6.28\ V_0 - 2.72\ V_1 - 4.04\ V_2 \ ;$$

Uniform depth h_m, $\Omega \neq 0$

$$(1.2) \quad \pi a^{-1} g h_m \zeta(\tfrac{1}{2}a, 0) = 1.31\ U_0 + 0.64\ U_1 + 0.19\ U_2 +$$
$$-6.28\ V_0 - 0.74\ V_1 - 3.14\ V_2 \ ;$$

Exponential depth, $\Omega = 0$

$$(1.3) \qquad \pi a^{-1} gh_m \, \zeta(\tfrac{1}{2}a, 0) = \qquad 0.51 \, U_1 + $$
$$-6.28 \, V_0 \qquad\qquad -4.04 \, V_2 \; ;$$

Uniform depth h_m, $\Omega = 0$

$$(1.4) \qquad \pi a^{-1} gh_m \, \zeta(\tfrac{1}{2}a, 0) = \qquad 0.26 \, U_1 + $$
$$-6.28 \, V_0 \qquad\qquad -3.14 \, V_2 \; .$$

The left-hand side indicates how these results must be interpreted if for a and h_m arbitrary values are chosen. In our case with a=400 km and h_m=65 m we have

$$(1.5) \qquad \pi a^{-1} gh_m = 5.0 \times 10^{-3} \; m/sec^2.$$

The relation between the absolute value of the frictional force of the wind and the velocity of the wind at sealevel is given by

$$(1.6) \qquad \sqrt{U^2 + V^2} = 3.0 \times 10^{-6} \, v_s^2.$$

By means of (1.5) and (1.6) the elevation can be found in meters (cf. also section II 6).

The general conclusion can be drawn that by the <u>combined</u> effect of the bottom slope (α) and the Coriolis force (Ω) the influence of the rotation terms of the windfield is greatly enhanced. The preliminary conclusions of the previous paper can now be given the more definite form

1. For a uniform N-S wind the elevation at the "Dutch" coast does not depend on the bottom profile nor on the rotation of the earth.

2. For a uniform W-E wind the elevation at the "Dutch" coast is much influenced by α and Ω.

3. For the given model the most unfavorable direction for a uniform wind as regards the elevation at the "Dutch" coast is about 15° NNW.

4. The influence of α and Ω upon the contributions of the divergence terms U_1 and V_2 to the elevation at the "Dutch" coast is rather small.

5. The influence of α and Ω upon the contributions of the rotation terms U_2 and V_1 to the elevation at the south coast is very large.

2. The mathematical problem

If the depth of the bay is given by the exponential law

(2.1)
$$h = h_o e^{2\alpha y}$$

the stationary state of the bay is determined by the equations

(2.2)
$$
\begin{cases}
\lambda u - \Omega v + c_o^2 e^{2\alpha y} \dfrac{\partial \zeta}{\partial x} = U \\[2mm]
\lambda v + \Omega u + c_o^2 e^{2\alpha y} \dfrac{\partial \zeta}{\partial y} = V \\[2mm]
\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0 ,
\end{cases}
$$

with the boundary conditions

(2.3)
$$
\begin{cases}
u = 0 & \text{at} \quad x = 0 \quad \text{and} \quad x = a \\
v = 0 & \text{at} \quad y = 0 \\
\zeta = 0 & \text{at} \quad y = b .
\end{cases}
$$

The meaning of c_o is as follows

(2.4)
$$c_o^2 \overset{def}{=} gh_o .$$

The harmonic mean h_m of the depth is given by

(2.5)
$$h_m = 2 \alpha b (1 - e^{-2\alpha b})^{-1} h_o ,$$

we shall also write

(2.6)
$$c_m^2 \overset{def}{=} gh_m ,$$

so that c_m represents the mean velocity of the free waves.

We shall take the same numerical case as in the previous paper viz.

$a = 400$ km	$\Omega = 0.44$ h^{-1}
$b = 800$ km	$\lambda = 0.09$ h^{-1}
$h_m = 65$ m	$\alpha = \pi/4b$.

Then according to (2.1) the depth increases from 33 m at the south coast to about 158 m at the ocean (see figure 2.1).

If x and y are measured in units of a/π the dimensionless values of a,b and α are respectively $a = \pi$, $b = 2\pi$, $\alpha = \dfrac{1}{8}$.

As in the previous paper we shall restrict ourselves to the discussion of the influence of a linear windfield of the following

type

$$(2.7) \begin{cases} U = U_0 + U_1 \ (1-2x/\pi) + U_2(1-y/b) \\ V = V_0 + V_1 \ (1-2x/\pi) + U_2(1-y/b). \end{cases}$$
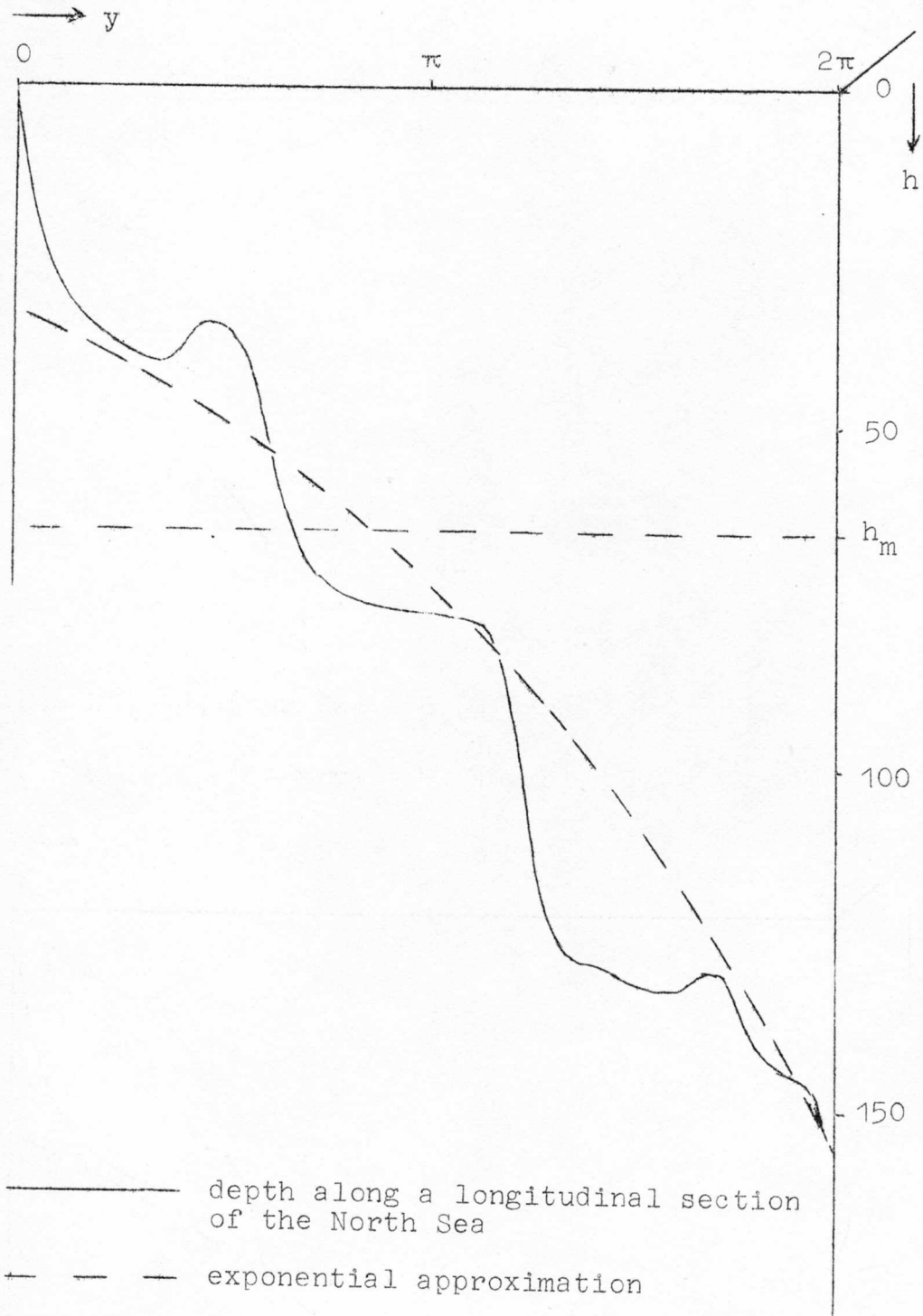

figure 2.1

3. A particular case

For the following windfield

(3.1) $\qquad U = 0 \qquad\qquad V = V(y)$

in the stationary state the stream is absent. In this case the quantities λ, Ω and h may even be arbitrary functions of y. The solution of the equations (2.2) is

(3.2) $\qquad u=v=0, \qquad g\,\zeta(x,y) = -\int_y^b h^{-1}(\eta)\,V(\eta)d\eta\;.$

Hence the stationary state appears to be independent of λ and Ω. If $V(y)$ is a constant, say $V=V_o$, the result (3.2) reduces to

(3.3) $\qquad\qquad\qquad gh_m\,\zeta(x,0) = -b\,V_o$

at the "Dutch" coast $y=0$. We note that for a sea of variable depth the same result is obtained as for a sea with the uniform harmonic mean depth $h=h_m$. For that reason the models considered in this and in the previous paper may be successfully compared to each other.

If the windfield (3.1) is linear,

(3.4) $\qquad U=0\;, \qquad V=V_o + V_2(1-y/b),$

the result (3.2) gives for $y=0$

(3.5) $\qquad gh_m\,\zeta(x,0) = -bV_o - b(1-y_m/b)V_2,$

where

(3.6) $\qquad\qquad\qquad y_m = \dfrac{h_m}{b}\int_0^b \dfrac{y}{h}\,dy.$

In the numerical case considered here we find

(3.7) $\qquad\qquad c_m^2\,\zeta(x,0) = -2\pi V_o - 4.04\,V_2\;.$

4. Method of solution

The treatment of the problem of the second section is very similar to that of the similar problem of section II 2 of the previous paper. Again a streamfunction $\phi(x,y)$ is introduced by means of

$$(4.1) \qquad \lambda u = - \frac{\partial \phi}{\partial y} \,, \qquad \lambda v = \frac{\partial \phi}{\partial x} \,.$$

From the equations (2.2) it follows by elimination of ζ that

$$(4.2) \qquad \lambda \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) + 2\alpha \lambda u - 2\alpha \Omega v = R + 2\alpha U,$$

where R represents the rotation of the windfield (cf. II 3.1)

$$(4.3) \qquad R = \frac{\partial V}{\partial x} - \frac{\partial U}{\partial y} \,,$$

Substitution of (4.1) in (4.2) gives a partial differential equation for the streamfunction

$$(4.4) \qquad \Delta \phi - 2A \frac{\partial \phi}{\partial x} - 2\alpha \frac{\partial \phi}{\partial y} = R + 2\alpha U,$$

where

$$(4.5) \qquad A \overset{\text{def}}{=} \alpha \Omega / \lambda \,.$$

The boundary conditions are

a the coast condition,

$$(4.6) \qquad \phi = 0 \qquad \text{at } x=0, \ x=\pi \,, \ y=0;$$

b the ocean condition,

$$(4.7) \qquad \frac{\partial \phi}{\partial y} + \text{tg}\,\mu\pi \ \frac{\partial \phi}{\partial x} = -U \qquad \text{at } y=b,$$

with

$$(4.8) \qquad \mu \overset{\text{def}}{=} \frac{1}{\pi} \, \text{arctg} \ \frac{\Omega}{\lambda} \,.$$

The problem of finding the streamfunction will be solved in the same way as in II 3. First a simple solution $\phi_o(x,y)$ satisfying the differential equation (4.4) and the boundary conditions at x=0 and x=π will be derived.

This function can be constructed as follows. We introduce the following two auxiliary functions which have the property of vanishing at x=0 and x=π

(4.9) $$\varphi_0(x) \overset{\text{def}}{=} \tfrac{1}{2}x(\pi-x),$$

(4.10) $$\varphi_1(x) \overset{\text{def}}{=} \frac{\pi}{2A}\left(\frac{e^{2Ax}-1}{e^{2A\pi}-1} - \frac{x}{\pi}\right).$$

If these functions are substituted in the left-hand side of (4.4) we find the following results

(4.11) $$\varphi_0(x) \rightarrow -(1+A\pi) + 2Ax$$

(4.12) $$\varphi_1(x) \longrightarrow 1.$$

Further we have

(4.13) $$x\ \varphi_1(x) \rightarrow \left(\frac{2\pi}{e^{2A\pi}-1} - \frac{1}{A}\right) + 3x + 2A\ \varphi_1(x),$$

(4.14) $$y\ \varphi_1(x) \longrightarrow y - 2\alpha\ \varphi_1(x).$$

Hence for $\phi_0(x,y)$ the following expression can easily be derived

(4.15) $$\phi_0(x,y) = (R + 2\alpha U_0)\varphi_1 - \frac{2\alpha U_1}{A\pi}(\varphi_0+\varphi_1) +$$
$$+ \frac{2\alpha U_2}{Ab}\left\{(A(b-y) - \alpha x)\ \varphi_1 + \frac{3\alpha}{2A}(\varphi_0+\varphi_1) + \right.$$
$$\left. + \frac{\alpha}{A}\ \frac{2A\pi}{e^{2A\pi}-1} - 1 + \frac{3}{2}A\pi)\ \varphi_1\right\}.$$

We put (cf. II 3.8)

(4.16) $$\phi(x,y) = \phi_0(x,y) + \phi_1(x,y)$$

so that ϕ_1 satisfies the homogeneous equation

(4.17) $$\Delta\phi_1 - 2A\ \frac{\partial\phi_1}{\partial x} - 2\alpha\ \frac{\partial\phi_1}{\partial y} = 0,$$

and the boundary conditions

(4.18) $$\phi_1 = 0 \qquad \text{at } x=0 \text{ and } x=\pi.$$

The elementary solutions of (4.17) and (4.18) are of the form (cf. II 3.11)

(4.19) $$e^{Ax} \sin nx\ e^{(\alpha\pm\mu_n)y}$$

for $n=1,2,3,\ldots$, where

(4.20) $$\mu_n \overset{\text{def}}{=} (n^2+A^2+\alpha^2)^{\frac{1}{2}}.$$

Hence we may put (cf. II 3.12)

(4.21)
$$\phi_1(x,y) = e^{Ax} \sum_{n=1}^{\infty} c_n \sin nx\, e^{-y(\mu_n - \alpha)} +$$
$$- e^{Ax} \sum_{n=1}^{\infty} d_n \sin nx\, e^{-(b-y)(\mu_n + \alpha)}.$$

The coast condition $\phi=0$ at $y=0$ gives by using (4.16) and (4.18)

(4.22)
$$\sum_{n=1}^{\infty} c_n \sin nx = -e^{-Ax}\, \phi_0(x,0) + O(e^{-b}),$$

where the order term contains the contributions of the terms with
the coefficients d_n (cf. II 3.18).
The coefficients c_n can be obtained by means of (4.22) and (4.15).
The explicit expression of c_n will not be written down in view of
its intricacy. We shall mention only its asymptotic behaviour, viz.

(4.23)
$$c_n = \frac{2}{\pi n^3} \left\{ R + 2\alpha(U_0 + U_1 + U_2) \right\} + (-1)^{n-1} \frac{2e^{-A\pi}}{\pi n^3} \left\{ R + 2\alpha(U_0 - U_1 + U_2) \right\} +$$
$$+ O(n^{-5}).$$

The ocean condition (4.7) gives a result that can be put in the
following form (cf. II 3.20)

(4.24)
$$\sum_{n=1}^{\infty} \frac{n\, d_n}{\mu_n - \alpha} \frac{d}{dx} \left\{ e^{Ax} \left(-\cos nx + \frac{\mu_n}{n} \operatorname{tg} \mu\pi \sin nx \right) \right\} =$$
$$= U(x,b) + \left(\frac{\partial}{\partial y} + \operatorname{tg}\mu\pi \frac{\partial}{\partial x} \right) \phi_0(x,b) + O(e^{-b}).$$

In some respects the integrated form of (4.24) has theoretical ad-
vantages.

(4.25)
$$\sum_{n=1}^{\infty} D_n(\sin nx - \theta_n \cos nx) = f(x) + O(e^{-b}),$$

where

(4.26)
$$D_n \overset{\text{def}}{=} \frac{\mu_n d_n}{\mu_n - \alpha},$$

(4.27)
$$\theta_n \overset{\text{def}}{=} \frac{n}{\mu_n} \operatorname{cotg} \mu\pi,$$

and

(4.28)
$$e^{Ax} f(x) = C + \operatorname{cotg}\mu\pi \int_0^x U(\xi,b)d\xi + \phi_0(x,b) +$$
$$+ \operatorname{cotg}\mu\pi \int_0^x \frac{\partial}{\partial y} \phi_0(\xi,b)d\xi,$$

where C is a constant of integration.

Since it follows from (4.27) and (4.20) that for $n \to \infty$

$$(4.29) \qquad \theta_n = \text{cotg}\, \mu\pi \left\{ 1 + O(n^{-2}) \right\}$$

the properties of the expansion (4.25) can be easily deduced from those of the simpler expansion

$$(4.30) \qquad \sum_{n=1}^{\infty} D_n \cos(nx + \mu\pi) = F(x), \qquad 0 < x < \pi,$$

where $F(x)$ may be considered a given function. The latter expansion has been considered in section II 4 of the previous paper. A more detailed treatment is given in Lauwerier (2). The main result is the asymptotic behaviour of D_n

$$(4.31) \qquad D_n = \frac{(-1)^{n-1} D'}{n^{2-2\mu}} + O(n^{-2-2\mu}),$$

where D' is a constant.

This result can be used to facilitate the numerical computation of the coefficients d_n. For the latter coefficients we find by means of (4.26)

$$(4.32) \qquad d_n = \frac{(-1)^{n-1} D'}{n^{2-2\mu}} + O(n^{-3+2\mu}).$$

We shall consider in particular the elevation of the sea at the coast $y=0$. In a similar way as in the previous paper the following formulae can be derived (cf. II 3.23 and II 3.25).

For the relative elevation at $y=0$ we have

$$(4.33) \qquad c_0^2 \{\zeta(x,0) - \zeta(0,0)\} = \int_0^x U(\zeta,0)d\zeta + \int_0^x \frac{\partial}{\partial y} \phi(\zeta,0)d\zeta .$$

Substitution of (4.15), (4.16) and (4.21) gives

$$(4.34) \quad c_0^2 \{\zeta(x,0) - \zeta(0,0)\} = \int_0^x U(\zeta,0)d\zeta - \frac{2\alpha}{b} U_2 \int_0^x \varphi_1(\zeta)d\zeta +$$

$$- \sum_{n=1}^{\infty} (\mu_n - \alpha)c_n \int_0^x e^{A\zeta} \sin n\zeta \, d\zeta + O(e^{-b}).$$

For the absolute elevation we have (cf. II 3.26)

$$(4.35) \qquad c_0^2 \, \zeta(0,0) = - \int_0^b e^{-2\alpha\eta} V(0,\eta)d\eta + \int_0^b e^{-2\alpha\eta} \frac{\partial}{\partial x} \phi_0(0,\eta)d\eta +$$

$$+ \sum_{n=1}^{\infty} \frac{n}{\mu_n + \alpha} c_n - e^{-2\alpha b} \sum_{n=1}^{\infty} \frac{n}{\mu_n - \alpha} d_n + O(e^{-b}).$$

5. The problem without rotation of the Earth

In order to get an impression of the influence of the rotation of the Earth the problem will also be solved with the assumption $\Omega=0$. The streamfunction is now determined by

$$(5.1) \qquad \Delta\phi - 2\alpha\frac{\partial\phi}{\partial y} = R + 2\alpha U,$$

and the boundary conditions

$$(5.2) \qquad \phi = 0 \quad \text{at} \quad x=0, \; x=\pi, \; y=0,$$

and

$$(5.3) \qquad \frac{\partial\phi}{\partial y} = -U \qquad \text{at} \qquad y=b.$$

We may put by specialisation of (4.16) and (4.21)

$$(5.4) \qquad \phi(x,y) = \phi_0(x,y) + \sum_{n=1}^{\infty} c_n \sin nx \; e^{-y(\mu_n-\alpha)} +$$
$$- \sum_{n=1}^{\infty} d_n \sin nx \; e^{-(b-y)(\mu_n+\alpha)},$$

where now

$$(5.5) \qquad \mu_n = (n^2+\alpha^2)^{\frac{1}{2}},$$

and

$$(5.6) \qquad \phi_0(x,y) = -\left\{ R + 2\alpha U_0 + \frac{2}{3}\alpha(1-\frac{2x}{\pi})U_1 + 2\alpha(1-\frac{y}{b})U_2 + \right.$$
$$\left. + \frac{2\alpha^2}{3b}(\varphi_0(x) + \frac{1}{2}\pi^2)\right\}\varphi_0(x).$$

The coast condition at $y=0$ gives by specialisation of (4.22)

$$(5.7) \qquad \sum_{n=1}^{\infty} c_n \sin nx = -\phi_0(x,0) + O(e^{-b}).$$

In order to find an explicit expression for c_n we need the following auxiliary integrals

$$(5.8) \qquad \int_0^{\pi} \varphi_0(x)\sin nx \; dx = \left\{1-(-1)^n\right\} n^{-3},$$

$$(5.9) \qquad \int_0^{\pi}(1-2x/\pi)\varphi_0(x)\sin nx \; dx = 3\left\{1+(-1)^n\right\}n^{-3},$$

$$(5.10) \qquad \int_0^{\pi}\left\{\varphi_0^2(x) + \frac{1}{2}\pi^2\varphi_0(x)\right\}\sin nx \; dx = 6\left\{1-(-1)^n\right\}n^{-5}.$$

Then with neglection of the order term it follows easily that

$$(5.11) \quad \begin{cases} c_n = \dfrac{8\alpha}{\pi n^3}\, U_1 \qquad \text{for even } n, \\[3mm] c_n = \dfrac{8\alpha}{\pi n^3}\, U_0 - \dfrac{8}{\pi^2 n^3}\, V_1 + \left\{ \left(2\alpha + \dfrac{1}{b}\right) \dfrac{4}{\pi n^3} + \dfrac{16\alpha^2}{b\pi n^5} \right\} U_2 \end{cases}$$

The ocean condition (5.3) gives

$$(5.12) \quad \sum_{n=1}^{\infty} (\mu_n + \alpha) d_n \sin nx = U_0 + U_1\left(1 - \frac{2x}{\pi}\right) + \frac{2\alpha}{b}\, U_2 \varphi_0(x) + O(e^{-b}),$$

from which, with neglection of the order term,

$$(5.13) \quad \begin{cases} (\mu_n + \alpha) d_n = \dfrac{4}{\pi n}\, U_1 \qquad \text{for even } n, \\[3mm] (\mu_n + \alpha) d_n = \dfrac{4}{\pi n}\, U_0 + \dfrac{8}{b\pi n^3}\, U_2 \quad \text{for odd } n. \end{cases}$$

The elevation at the "Dutch" coast is determined by (4.34) with $A=0$ and by (4.35). We shall give here only the explicit expression for the elevation at "Hoek van Holland", the middle of the "Dutch" coast.

$$(5.14) \quad c_0^2\, \mathfrak{z}(\tfrac{1}{2}\pi, 0) = \left\{ \frac{\pi}{12} + \frac{8\alpha}{\pi} \sum_{n=1}^{\infty} \frac{\mu_n - \alpha}{n^4} \cos \tfrac{1}{2} n\pi \right\} U_1 +$$

$$- \frac{h_0}{h_m} \left\{ 2\pi V_0 + 4.04\, V_2 \right\} + O(e^{-b}).$$

We note that the contributions of the components U_0, U_2 and V_1 vanish at this point.

6. Numerical application

The calculation of the elevation at the "Dutch" coast due to the linear windfield (2.7) may be carried out with the help of (4.34) and (4.35).

In the first place we need the coefficients c_n. They are linear expressions in U_0, U_1, U_2 and V_1 the factors of which are given in the following table for a few values of n.

	U_0	U_1	U_2	V_1
c_1	0,094	0,017	0,154	-0,239
c_2	0,014	0,016	0,022	-0,036
c_3	0,006	0,004	0,010	-0,016
c_4	0,002	0,003	0,003	-0,005
c_5	0,001	0,001	0,002	-0,004

table 6.1

The calculation of the relative elevation by means of (4.34) gives the following result

$8x/\pi$	C_0	C_1	C_2	D_0	D_1	D_2
0	0	0	0	0	0	0
1	0,37	0,33	0,36	0	0,05	0
2	0,70	0,54	0,66	0	0,21	0
3	1,02	0,66	0,93	0	0,40	0
4	1,33	0,69	1,21	0	0,62	0
5	1,63	0,64	1,46	0	0,86	0
6	1,93	0,52	1,73	0	1,06	0
7	2,24	0,29	1,99	0	1,31	0
8	2,59	-0,04	2,33	0	1,40	0

table 6.2

where

$$(6.1) \quad c_0^2 \{ \zeta(x,0) - \zeta(0,0) \} = C_0 U_0 + C_1 U_1 + C_2 U_2 +$$
$$+ D_0 V_0 + D_1 V_1 + D_2 V_2 .$$

For the absolute elevation, which is given by (4.35), we write

(6.2) $$c_o^2 \mathfrak{z}(0,0) = I + II + III,$$

where

(6.3) $$I = -\int_0^b e^{-2\alpha\eta} V(0,\eta)d\eta + \int_0^b e^{-2\alpha\eta} \frac{\partial}{\partial x} \phi_o(0,\eta)d\eta \, ,$$

(6.4) $$II = \sum_{n=1}^{\infty} \frac{n}{\mu_n+\alpha} c_n \, ,$$

(6.5) $$III = -e^{-2\alpha b} \sum_{n=1}^{\infty} \frac{n}{\mu_n-\alpha} d_n \, .$$

The calculation of the expressions I and II is simple and straight-forward. The calculation of III, however, requires the calculation of the coefficients d_n which is a rather difficult problem. We shall not give a detailed description of the numerical process, but we restrict ourselves to the remark that much profit has been obtained from the asymptotic expressions (4.31) and (4.32). The results of the calculation are given below, however without the contribution of the components V_o and V_2.

	U_o	U_1	U_2	V_1
I	-0,58	-0,34	-0,67	-1,68
II	0,10	0,04	0,17	-0,26
III	-0,01	-0,01	0,01	-0,05

table 6.3

We note that the contribution from III is very small. From this table and from (3.7) it follows that

(6.6) $$-c_o^2 \mathfrak{z}(0,0) = 0.49 \, U_o + 0.31 \, U_1 + 0.49 \, U_2 +$$
$$+ 3.17 \, V_o + 1.99 \, V_1 + 2.04 \, V_2 \, .$$

The results of table 6.2 and formula (6.6) may be combined in order to give the absolute elevation $c_m^2 \mathfrak{z}(x,0)$.

$8x/\pi$	C_0	C_1	C_2	D_0	D_1	D_2
0	-0,97	-0,61	-0,97	-6,28	-3,95	-4,04
1	-0,24	0,04	-0,26	"	-3,85	"
2	0,42	0,46	0,34	"	-3,53	"
3	1,05	0,69	0,88	"	-3,15	"
4	1,67	0,71	1,43	"	-2,72	"
5	2,26	0,65	1,92	"	-2,24	"
6	2,86	0,42	2,46	"	-1,84	"
7	3,47	-0,04	2,97	"	-1,35	"
8	4,16	-0,69	3,65	"	-1,17	"

table 6.4

where now

$$(6.7) \qquad c_m^2 \; \zeta(x,0) = C_0 U_0 + C_1 U_1 + C_2 U_2 +$$
$$+ D_0 V_0 + D_1 V_1 + D_2 V_2 \; .$$

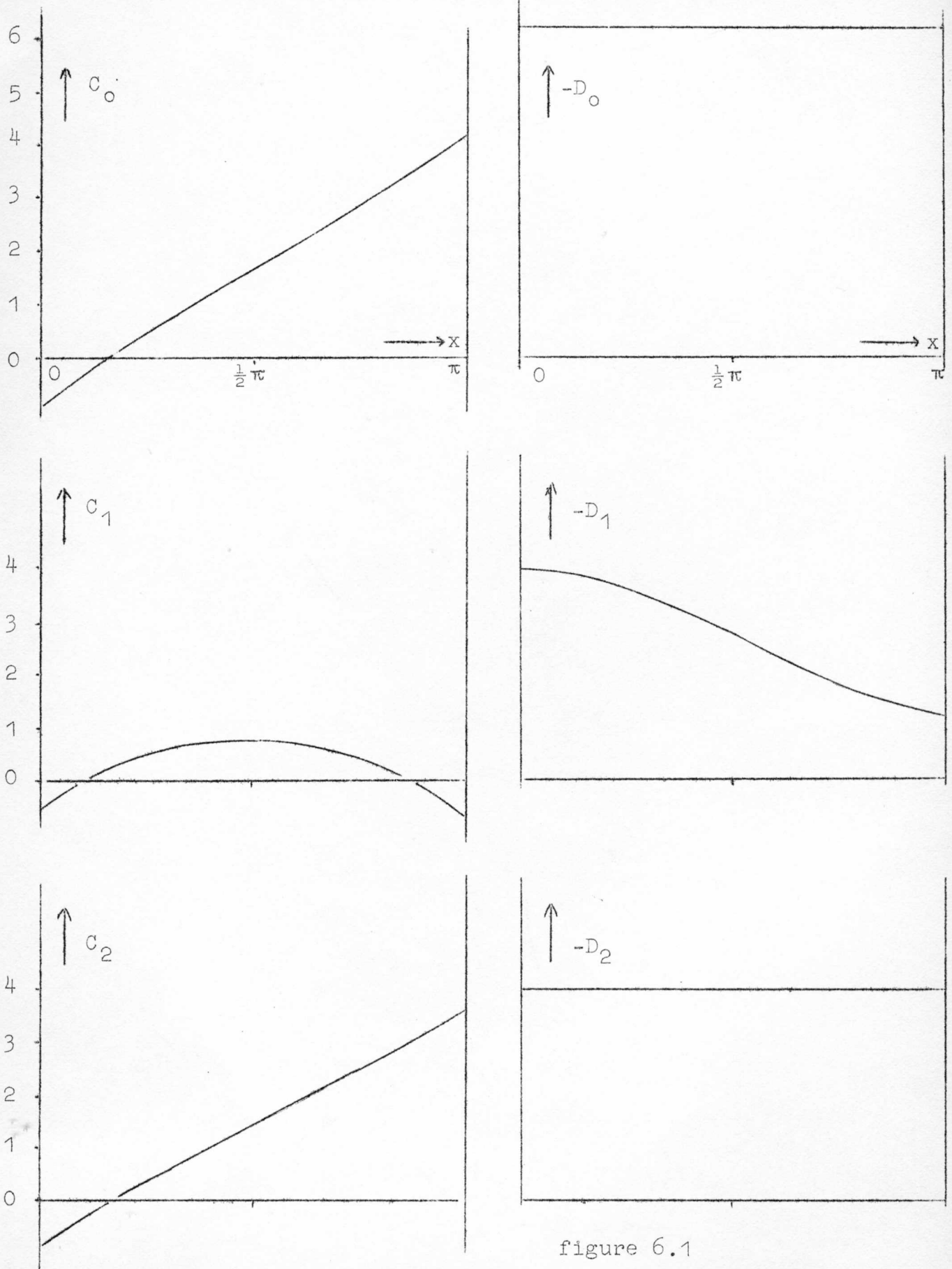A graphical illustration of this table is given in figure 6.1

figure 6.1

Elevation at the "Dutch" coast due to a linear windfield

The calculation of the elevation with absence of the rotation of the Earth can be carried out by means of (4.34) and (5.14). With the notation of (6.7) the following table can be constructed

$8x/\pi$	C_0	C_1	C_2	D_0	D_1	D_2
0	-2.54	-0.89	-2.26	-6.28	-1.49	-4.04
1	-1.82	-0.26	-1.57	"	-1.31	"
2	-1.19	0.18	-1.01	"	-0.95	"
3	-0.59	0.44	-0.52	"	-0.50	"
4	0	0.52	0	"	0	"
5	0.59	0.44	0.52	"	0.50	"
6	1.19	0.18	1.01	"	0.95	"
7	1.82	-0.26	1.57	"	1.31	"
8	2.54	-0.89	2.26	"	1.49	"

table 6.5

Literature

See the bibliography at the end of the previous paper.